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Reg. No. : ....

Name : ....

Fifth Semester B.Tech. Degree Examination, November 2013
(2008 Scheme)
08.501 : ENGINEERING MATHEMATICS – IV
Complex Analysis and Linear Algebra (T A)

Time : 3 Hours Max. Marks : 100

Answer **all** questions from Part **A** and **one full** question from **each** Module of Part **B**.

PART – A

1. Determine the region of the complex plane where CR equations are satisfied for the function $f(z) = (x - y)^2 + 2i(x + y)$.

2. Show that an analytic function whose modulus is constant, is a constant function.

3. Find the harmonic conjugate of $u = 3xy^2 - x^3$.

4. Find the image of the rectangular region

   $-1 \le x \le 3, \quad -\pi \le y \le \pi$ in the z plane under the mapping $w = e^z$.

5. Define fixed points of a transformation. Find the fixed points of $w = \frac{2z-5}{z+4}$.

6. Evaluate $\oint_c \frac{e^{3z}dz}{(z-\ln(2))^4}$, where C is the square with vertices $\pm 1, \pm i$.

7. Expand $f(z) = \frac{1}{1+z^2}$ in a Tayalors series about $z = 0$.

P.T.O.

8. Define basis of a vector space. Is $u_1 = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$ $u_2 = \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$ $u_3 = \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix}$ a basis of $\mathbb{R}^3$.

9. Show that the set nul A = $\{x \in R^n : Ax = 0\}$ is a subspace of $\mathbb{R}^n$ where A is an $m \times n$ matrix.

10. Find the singular values of the matrix $A = \begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}$.

PART – B

Module – I

11. a) Show that $f(z) = Re(z) = x$ is continuous but not differentiable.

b) If f(z) is analytic show that $\left(\dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2}\right) |Re\ f(z)|^2 = 2|f'(z)|^2$.

c) Find the bilinear transformation that maps i, –i, 1 onto 0, 1, ∞ respectively.

12. a) Show that log z is differentiable except at z = 0 and find its derivative.

b) Show that for $f(z) = \dfrac{2xy(x+iy)}{x^2+y^2}$; $z \neq 0$

$= 0 \; ; \; z = 0$

CR equations are satisfied at the origin, but derivative of f(z) at the origin doesn't exist.

c) Find the image of the strip $1 \leq y \leq 3/2$ under $W = z^2$.

Module – II

13. a) Evaluate $\oint_c \frac{dz}{z^2+9}$ where C is

i) $|z-3i| = 4$

ii) $|z+3i| = 2$.

b) Find the Laurent's series of $f(z) = \frac{7z^2+9z-18}{z^3-9z}$ in i) $0 < |z| < 3$ ii) $|z| > 3$.

c) Determine and classify the singular points of $\frac{1}{(2\sin z-1)^2}$.

14. a) Determine the residues at the poles of $f(z) = \frac{2z+1}{z^2-z-2}$

b) Evaluate $\int_0^{2\pi} \frac{d\theta}{\sqrt{2}-\cos\theta}$

c) Evaluate $\int_0^{\infty} \frac{dx}{(a^2+x^2)^2}$

Module – III

15. a) Find the LU decomposition of $A = \begin{bmatrix} 2 & -1 & 2 \\ -6 & 0 & -2 \\ 8 & -1 & 5 \end{bmatrix}$ and use it to solve

$AX = B$, where $B = \begin{pmatrix} 1 \\ 0 \\ 4 \end{pmatrix}$.

b) Find the null space and column space of an nxn non-singular matrix.

c) Explain Gram-Schmidt process. Use it to find an orthonormal basis of R^3

from $v_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$ $v_2 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ $v_3 = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$

16. a) Find the least square solution of AX = B where

$$A = \begin{bmatrix} 1 & 2 \\ -1 & 1 \\ 1 & 3 \end{bmatrix} \qquad B = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

b) Show that the quadratic form X^TAX is positive definite if and only if the eigen values of A are all positive.

c) Obtain the singular value decomposition of $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 0 & 0 \end{bmatrix}$